MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2018/2019

EMT1016 – ENGINEERING MATHEMATICS I

(All Sections / Groups)

6 MARCH 2019

2.30p.m – 4.30p.m

(2 Hours)

INSTRUCTIONS TO STUDENTS:

1. This exam paper consists of **4 pages** (including cover page) with **4** Questions only.
2. Attempt **all** 4 questions. All questions carry equal marks and the distribution of marks for each question is given.
3. Please print all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.

Question 1

(a) Evaluate the following limits, if they exist.

(i) $\lim\limits_{x\to-\infty} \dfrac{\sqrt{9x^2-2}}{x+3}$ [4 marks]

(ii) $\lim\limits_{x\to\infty} x^{\frac{1}{x}}$ [5 marks]

(b) A function is given below:

$$f(x)=\begin{cases} 3-x, & 0<x\le 3, \\ (x-3)^2, & x>3. \end{cases}$$

(i) Perform continuity test for $f(x)$ at $x=3$. Is $f(x)$ continuous at $x=3$? [5 marks]

(ii) Sketch $f(x)$. Does the inverse function exist for $f(x)$? Provide your justification. [3 marks]

(c) Use **partial fraction decomposition** to find $\int \dfrac{1}{x(x^2-9)}dx$. [8 marks]

Question 2

(a) Let $x^2+z\sin(xyz)=1$. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ using **implicit differentiation.** [6 marks]

(b) The radius r (in cm), volume V (in cm^3) and height h (in cm) of a right circular cylinder are related by the equation $V=\pi r^2 h$. Use **chain rule** to find $\dfrac{\partial V}{\partial t}$, which is the rate at which the volume is changing when the radius is 2cm and increasing at a rate of 0.4cm/min, and the height is 5cm and decreasing at a rate of 0.3 cm/min. [7 marks]

(c) Let $f(x,y,z)=x+y+2z$. Use the method of **Lagrange multipliers** to find the maxima and minima of f on the surface $x^2+y^2+z^2=3$. [12 marks]

Continued...

Question 3

(a) Given a complex number $z = 2 + i4$. Find the real constants a and b that satisfy

$$\frac{z+\bar{z}}{\bar{z}-z} = a + ib.$$

[5 marks]

(b) Find all the four complex roots of the equation $z^4 - 8 - 8\sqrt{3}i = 0$.

[8 marks]

(c) Use the Maclaurin series $\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ to expand $f(x) = x^3 \sin(2x)$. Then, give the first four terms.

[6 marks]

(d) Find the radius of convergence of the power series $\sum_{n=0}^{\infty} (-1)^n \frac{x^n}{(2n^2+3)}$.

[6 marks]

Question 4

(a) Complete the following statements:

(i) Suppose $g(x)$ and $h(x)$ are periodic functions. If $g(x)$ has a period of 4π and $h(x)$ has a period of 6π, then $g(x)+h(x)$ has a period of ________.

(ii) If $g(x)$ is an even periodic function and $h(x)$ is an odd periodic function, then the product $g(x) \cdot h(x)$ is an ________ periodic function.

(iii) The Fourier series

$$\frac{12}{\pi}\sum_{k=1}^{\infty} \frac{1}{2k-1} \sin\frac{(2k-1)\pi x}{6}$$

represents a periodic function that has a period of __________ and is symmetrical about the __________.

(iv) A Fourier series expansion of $g(x)$ is guaranteed to converge to $g(x)$ if __________ are satisfied.

[5 marks]

Continued...

(b) A periodic function $f(x)$ of **period 2π** is defined over $(-\pi, \pi]$ by

$$f(x)=\begin{cases}0, & \text{if } \quad -\pi < x \le 0,\\ 1, & \text{if } \quad 0 < x \le \pi.\end{cases}$$

(i) Sketch the graph of $f(x)$ from $x=-2\pi$ to $x=2\pi$.

[4 marks]

(ii) Compute the Fourier coefficients (i.e., a_0, a_n and b_n) of $f(x)$. Then, write its Fourier series expansion.

[16 marks]

End of paper.